Determining the Relevant Fair Value(s) of S&P 500 Futures

By Ira G.Kawaller

A fundamental consideration for potential users of stock index futures is the determination of the futures' break-even price or fair value. Conceptually, being able to sell futures at prices above the break-even or buy futures at prices below the break-even offers opportunity for incremental gain. This article points out an important, though widely unappreciated caveat. That is, no single break-even price is universally appropriate. Put another way, the break-even price for a given institution depends on the motivation of that firm as well as its marginal funding and investing yield alternatives.

In this article five differentiated objectives are identified, and the calculations of the respective break-even futures prices are provided. The various objectives are: (a) to generate profits from arbitrage activities, (b) to create synthetic money market instruments, (c) to reduce exposure to equities, (d) to increase equity exposure and (e) to maintain equity exposure using the more cost effective instrument via stock/futures substitution.

All these alternative objectives have the same conceptual starting point, which relates to the fact that a combined long stock/short futures position generates a money market return composed of the dividends on the stock position as well as the basis[1] adjustment of the futures contract. Under the simplified assumptions of zero transaction costs and equal marginal borrowing and lending rates, the underlying spot/futures relationship can be expressed as follows:

(1) $$F = S\left(1 + (i - d)\frac{t}{360}\right)$$

Where F = break-even futures price

S = spot index price

i = interest rate (expressed as a money market yield)

d = projected dividend rate (expressed as a money market yield)

t = number of days from today's spot value date to the value date of the futures contract.

In equilibrium, the actual futures price equals the break-even futures price, and thus the market participant would either have no incentive to undertake the transactions or be indifferent between competing tactics for an equivalent goal.

Moving from the conceptual to the practical simply requires the selection of the appropriate marginal interest rate for the participant in question, as well as precise accounting for transaction costs. This paper demonstrates that these considerations foster differences between the break-even prices among the alternative goals considered. Each goal is explained more fully, and the respective theoretical futures prices are presented.

[1] "Basis" in this paper is defined as the futures price minus the spot index value. Elsewhere, the calculation might be made with the two prices reversed.

Generating Profits from Arbitrage Activities

Generally, arbitrage requires identifying two distinct marketplaces where something is traded, and then waiting for opportunities to buy in one market at one price and sell in the other market at a higher price. This same process is at work for stock/futures arbitrage, but these market participants tend to view their activities with a slightly different slant. They will enter an arbitrage trade whenever (a) buying stock and selling futures generates a return that exceeds financing costs, or (b) selling stock and buying futures results in an effective yield (cost of borrowing) that falls below marginal lending rates. Completed arbitrages will require a reversal of the starting positions, and the costs of both buying and selling stocks and futures must be included in the calculations.[2] Thus, the total costs of an arbitrage trade reflects the bid/ask spreads on all of the stocks involved in the arbitrage, the bid/ask spreads for all futures positions, and all commission charges on both stocks and futures.[3]

Table 1 (page 3) calculates these arbitrage costs under three different scenarios. In all cases, the current starting value of the stock portfolio, based on last-sale prices, is $100 million and the S&P 500 index is valued at 950.00. The size of the hedge ratio is calculated in the traditional manner:[4]

(2) $$H = \frac{V \times Beta}{S\&P \times 250}$$

Where H = size of the hedge (number of futures contracts required)

V = value of the portfolio

Beta = portfolio beta

S&P = spot S&P 500 index price

250 = the multiplier on the S&P 500 index futures contract, and the average price per share is estimated to be $50.

In column A, transactions are assumed to be costless, reflected by zero values for bid/ask spreads as well as zero commissions. In column B, more typical conditions are shown. Commissions on stock are assumed to be $.02 per share; bid/ask spreads on stocks are assumed to be 1/8 ($.125 per share); commissions on futures are assumed to be $12 on a round-turn basis (i.e., for both buy and sell transactions); and bid/ask spreads on futures are assumed to be two ticks or 0.20, worth $50. Column C assumes the same commission structure as that of column B, but bid/ask spreads are somewhat higher, reflecting a decline in liquidity relative to the former case. This scenario also might be viewed as representing the case where impact costs of trying to execute a stock portfolio were expected to move initial bids or offers for a complete execution. The index point costs in all cases reflect the respective dollar costs on a per-contract basis.[5]

The arbitrageur would evaluate two independent arbitrage bounds: An upper bound and a lower bound. During those times when futures prices exceed the upper arbitrage boundary, profit could be made by financing the purchase of stocks at the marginal borrowing rate and selling futures; and when the futures prices are below the lower bound, profits could be made by selling stocks and buying futures, thus creating a synthetic borrowing, and investing at the marginal lending rate. In both cases, the completed arbitrages would require an unwinding of all the original trades.

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2 If any fees or charges apply to the borrowing or lending mechanisms, these, too, would have to be incorporated in the calculations. Put another way, for the calculations that are presented in this article, the marginal borrowing and lending rates are effective rates, inclusive of all such fees.

3 Brennan & Schwartz (1990) note that the cost of closing an arbitrage position may differ if the action is taken at expiration versus prior to expiration. Thus, the appropriate arbitrage bound should reflect whether or not the arbitrageur is expecting (or hoping) to exercise an "early close-out option."

4 See Kawaller (1985) for a discussion of the justification for this hedge ratio.

5 In practice, it may be appropriate to assume two different cost structures for the upper- and lower-bound break-even calculations, because costs differ depending on whether the trade starts with long stock/short futures or vice versa. The difference arises because initiating the short stock/long futures arbitrage requires the sale of stock on an uptick. The "cost" of 0this requirement is uncertain because the transactions price is not known at the time the decision is made to enter the arbitrage. No analogous uncertainty exists when initiating the arbitrage in the opposite direction.

TABLE 1: Arbitrage Costs

	A	B	C
S&P 500 Index Value	950.00	950.00	950.00
Size of Portfolio	100,000,000	$100,000,000	$100,000,000
Average Price per Share	50.00	50.00	50.00
Number of Shares	2,000,000	2,000,000	2,000,000
Commission per Share of Stock	0.00	0.02	0.02
Stock Commissions per Side	0.00	40,000	40,000
Stock Commissions (RT)	0.00	80,000	80,000
Bid/Ask Per Unit of Stock	0.00	0.125	0.50
Bid/Ask Stock	0.00	250,000	1,000,000
Contracts	421	421	421
Commissions per Round Turn	0.00	12.00	12.00
Futures Commissions	0.00	5,052	5,052
Bid/Ask per Futures Contract	0.00	0.20	1.00
Bid/Ask Futures	0.00	21,050	105,250
Dollar Costs	0.00	$356,102	$1,190,302
Index Point Cost per Futures Contract	0.00	3.38	11.31
Marginal Borrowing Rate	6.00%	6.00%	6.00%
Marginal Lending Rate	5.00%	5.00%	5.00%
Dividend Rate	3.50%	3.50%	3.50%
Shorter Horizon (Case a):			
Days to Expiration	30	30	30
Upper Bound	951.98	955.36	963.29
Lower Bound	951.19	947.80	939.88
No-Arbitrage Range	0.79	7.56	23.41
Longer Horizon (Case b):			
Days to Expiration	60	60	60
Upper Bound	953.96	957.34	965.27
Lower Bound	952.38	948.99	941.07
No arbitrage range	1.58	8.35	24.20

The upper bound is found by substituting the arbitrage firm's marginal borrowing rate in equation (1) and adding the arbitrage costs (in basis points) to this calculated value. In the case of the lower arbitrage boundary, the marginal lending rate is used for the variable *i* in equation (1), and the arbitrage costs are subtracted. The calculations in Table 1 assume marginal borrowing and lending rates of 6% and 5%, respectively, and a dividend rate of 3.5%. The upper and lower arbitrage boundaries are given for the three alternative cost structures. For comparative purposes, arbitrage boundaries are generated for two different time periods.

Most obvious is the conclusion that an arbitrageur with a higher (lower) cost structure or a wider (narrower) differential between marginal borrowing and lending costs would face wider (narrower) no-arbitrage boundaries. In addition, Table 1 also demonstrates the time-sensitive nature of the difference between the two bounds, or the no-arbitrage range. As time to expiration expands, this range increases monotonically, all other considerations held constant.

Creating Synthetic Money Market Securities

The case of the firm seeking to construct a synthetic money market security by buying stocks and selling futures is a slight variant of the arbitrage case described in the prior section.[6] In this situation, too, the firm will seek to realize a rate of return for the combined long stock/short futures positions, but the relevant interest rate that underlies the determination of the break-even futures price is different. While the arbitrageur who buys stock and sells futures will do so whenever the resulting gain betters the marginal *borrowing* rate, the synthetic fixed-income trader will endeavor to outperform the marginal *lending* rate. For both, however, the imposition of transaction costs will necessitate the sale of the futures at a higher price than would be dictated by the costless case.

Not surprisingly, the break-even price for this player is directly related to both transaction costs and time to expiration. What may not be quite as readily apparent is the fact that, at least theoretically, situations may arise that provide no motivation for arbitrageurs to be sellers of futures, while at the same time offering a motivation for a potentially much larger audience of money managers to be futures sellers. Put another way, large scale implementation of the synthetic money market strategy by many market investors could certainly enhance these participants' returns,but also have the more universally beneficial effect of reducing the range of futures price fluctuation that do not induce relative-price-based trading strategies.

Yet another seemingly perverse condition that is highlighted by these calculations is that firms that operate less aggressively in the cash market, and thereby tend to have lower marginal lending rates, will likely have a greater incremental benefit from arranging synthetic securities than will firms that seek out higher cash market returns. For example, assume Firm A has access to Euro deposit markets while Firm B deals only with lower yielding U.S. domestic banks; and assume further that the difference in marginal lending rates is 0.25%. Firm B's break-even futures price necessarily falls below that of Firm A. At any point in time, however, the current futures bid is relevant for both firms. Assuming the two firms faced the same transaction cost structures, this futures price would generate the same effective yield for the two firms. Invariably, Firm B will find a greater number of yield enhancement opportunities than will Firm A; and any time both firms are attracted to this strategy simultaneously, B's incremental gain will be greater.

Decreasing Equity Exposure

The case of the portfolio manager who owns equities and is looking to eliminate that exposure has two alternative courses of action: He/she could (1) simply sell the stocks, or (2) continue to hold the equities and overlay a short futures position. If the adjustment were expected to be permanent, the first course of action would likely be preferred, as the stocks would have to be liquidated anyway, at some point.Thus, the use of a futures hedge would only delay the inevitable and add additional costs.When the adjustment to the equity exposure is expected to be temporary, on the other hand, the use of futures would likely make more sense, given the significantly lower transactions costs associated with the use of futures versus traditional shares. Even in this case, however, there is a break-even futures price below which the short futures hedge becomes uneconomic, despite the advantageous transaction cost comparison.

This break-even price is found by recognizing that the effect of the hedge is to convert the equity exposure into a money market return.

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[6] The case where the firm already holds the stock is considered later.

TABLE 2: Reducing Equity Exposure

	Temporary Shift Out of Equities
Index Value	950.00
Size of Portfolio	$100,000,000
Portfolio Beta	1.00
Avg Price per Share	50.00
No. of Shares	2,000,000
Commission per Share	0.02
Commissions per Side	40,000
Bid/Ask per Stock	0.125
1/2 Bid/Ask Stock	125,000
Total Stock Costs per Side	165,000
Investable Funds	99,835,000
Marginal Lending Rate	5.00%
Hedge Calculation	421.1
Contracts	421
Commissions (Rnd Trn)	12.00
Futures Commissions	5,052
Bid/Ask per Contract	0.20
Bid/Ask Futures	21,050
Total Futures Costs	236,102
Futures Costs/Contract ($)	62
Futures Costs (Index Points)	0.25
Dividend Rate	3.50%

Shorter Horizon (Case a):		***Longer Horizon (Case b):***	
Days to End Point	30	Days to End Point	60
Ending Value	$100,085,979	Ending Value	$100,501,958
Effective Money Market Return	1.03%	Effective Money Market Return	3.01%
Dividend Income	291,667	Dividend Income	583,333
Residual Income Required	(205,687)	Residual Income Required	(81,375)
Break-even Futures Price	948.29	Break-even Futures Price	949.47

The question then becomes,"What is the effective money market return that one could realize by selling the stocks,putting the funds in a money market security, and then repurchasing the stocks?" Given this result, one must then find the futures price that generates this same result. Clearly, if the futures could be sold at a higher price, the short hedge would be the preferred way to decrease the equity exposure.

In calculating the returns from the traditional "sell stocks/buy money market securities" tactic, one should recognize that the liquidation cost effectively "haircuts" the portfolio. For example, the liquidation of a $100 million portfolio involves an immediate expense such that some amount less than the original $100 million becomes available for reinvestment. Thus, the portfolio manager realizes a lower fixed income return than the nominal yield on the proposed money market security.

In Table 2 (page 5), the haircut is estimated to reflect half of the bid/ask spread as well as the stock commissions. The same commission and bid/ask structure is assumed as that which faces the firms analyzed in the prior section; and similarly, the same marginal investment rate is incorporated. Under these conditions, the manager who chooses the liquidation of the stock portfolio and the investment of the proceeds at 5% (rather than hedging) realizes an effective net money market return of 1.03% for 30 days or 3.01% for 60 days. Respective break-even futures prices are 948.29 and 949.47. Implicit in these calculations is the assumption that the period for which the funds will remain in cash is identically equal to the horizon associated with the money market instrument and the time to the expiration of the futures contract when futures and spot prices necessarily converge.

Increasing Equity Exposure

Perhaps the easiest situation to explain is the choice between buying equities today at the spot price versus leaving the fund in a money market instrument and entering a long futures position. This decision simply requires calculating the forward value of the index which, in turn, reflects the opportunity costs of foregoing interest income of a fixed income investment alternative as well as an adjustment for transaction costs of futures, alone.[7] For the case of the same prototype firm discussed in the earlier sections, and given the same portfolio, the opportunity cost is generated using the marginal lending rate of 5%. This value corresponds to the lower arbitrage boundary in the zero transactions cost scenario.

Futures costs total $26,102 (= $5,052 + $21,050)

or about $62 (= $\frac{\$26{,}102}{421\ \textit{contracts}}$) per contract.

In terms of an adjustment to the futures price, $62 represents a price effect of about 0.25 or two-and-a-half ticks. Thus, in this case, with the spot S&P 500 index at 950.00 and 30 days to the futures value date, the break-even price is 950.94 (= 951.19 – 0.25). For a 60-day horizon, the break-even becomes 952.13 (= 952.38 – 0.25).

Maintaining Equity Exposure in the Most Cost-Effective Instrument

Consider the case of the portfolio manager who currently holds equities, with the existing degree of exposure at the desired level. Even this player may find using futures to be attractive if they are sufficiently cheap. At some futures price it becomes attractive to sell the stocks and buy the futures, thereby maintaining the same equity exposure. The break-even price for this trader, then, would be the trigger price. That is, any futures price lower than this break-even would induce the substitution of futures for stocks and generate incremental benefits.

Like the prior case, this strategy rests on the comparison of present versus future values; and again, the firm's marginal lending rate is the appropriate discounting factor. Regarding trading costs, commissions and bid/ask spreads for both stocks and futures must be taken into account, as the move from stocks to futures would be temporary. Thus, the break-even price would be lower than the zero-cost theoretical futures price by the basis point costs of the combined commissions and bid/ask spreads.

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[7] Stock costs would be roughly comparable whether one were to buy now or later, so they do not enter into the calculation. This treatment, admittedly, is not precise. For example, with a significant market move, the number of shares required may vary, as may the average bid-ask spreads; therefore, some differences may arise. Moreover, the statement ignores the fact that although absolute magnitudes may be identical in both the buy-now or buy-later cases, the present values of these charges may differ. This consideration, if taken into account more rigorously, would bias the decision toward a later purchase. For the purposes of this analysis, however, these differences are ignored.

[8] This result happens to be identical to that shown for the lower arbitrage bound of the firm operating with the same cost structure. As explained in footnote 5, however, the arbitrage firm that sells stock short has additional costs that do not apply to the stock/futures substituter. Thus, in practice, the break-even for the substituter is likely to be a higher price than the lower bound for the equivalent firm involved with arbitrage.

TABLE 3: Alternate Break-even Prices

	Days to Expiration	
	30 Days	60 Days
Lower Arbitrage Boundary*/Futures Substitution Break-even	947.80	948.99
Long Hedge Break-even	950.94	952.13
Short Hedge Break-even	948.29	949.47
Synthetic Fixed Income Break-even	954.57	955.76
Upper Arbitrage Boundary	955.36	957.34

*Not reflective of costs associated with the uptick rule.

For the prototype firm with the marginal lending rate of 5%, under the same normal market assumptions used throughout, the break-even price for 30-and-60-day horizons becomes 947.80 and 948.99, respectively.[8]

Consolidation and Summary

In Table 3, the respective break-even prices that are relevant to the various applications discussed in the article are shown. All calculations relate to a firm with a marginal borrowing rate of 6% and a marginal lending rate of 5%. Break-even prices are given for two different time spans for the hedging period: 30 days and 60 days. Further, these calculations reflect the additional assumption of "normal" transaction costs and bid/ask spreads.

The highest price for which it becomes advantageous to take a long futures position is the long hedger's break-even price; and if prices decline sufficiently from this value, such that they fall below the lower arbitrage boundary, additional market participants — namely arbitrageurs — would be induced to buy futures,as well. The lowest price for which it becomes advantageous to sell futures would be the break-even for the temporary short hedger; and in a similar fashion if prices rise sufficiently above this level,additional short sellers would be attracted to these markets.

Note that regardless of the time horizon, the maximum price for which buying futures is justified (950.94 or 952.13) is higher than the lowest price for which selling futures is justified (948.29 or 949.47). Thus, at every futures price there is at least one market participant who "should" be using this market. Moreover, it is also interesting that if the futures price enables the arbitrageur to operate profitably, at least one other market participant would find the futures to be attractively priced as well. For example, if the futures price were below the lower arbitrage bound, aside from the arbitrageur, the long hedger would certainly be predisposed to buying futures rather than buying stocks; and if the futures price were above the upper arbitrage bound, willing sellers would include arbitrageurs, short hedgers, and those constructing fixed income securities.

The overall conclusion, then, is that it pays (literally) to evaluate the relevant break-even prices for any firm interested in any of the above strategies — a population that includes all firms that manage money market or equity portfolios. At every point in time, at least one strategy will dictate the use of futures as the preferred transactions vehicle, because use of futures in this situation will add incremental value. Failure to make this evaluation will undoubtedly result in either using futures at inopportune moments, or more likely failing to use futures when it would be desirable to do so. In either case, neglecting to compare the currently available futures price to the correct break-even price will ultimately result in suboptimal performance.

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Ira G.Kawaller is president of Kawaller & Company, LLC, located in Brooklyn,N.Y. — a financial consulting firm specializing in risk management tools,strategies, and techniques.He serves as a senior consultant to the Chicago Mercantile Exchange.

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